

Discrete Optimization for Optical Flow

Moritz Menze¹ Christian Heipke¹ Andreas Geiger²

¹Institute of Photogrammetry and GeoInformation
Leibniz Universität Hannover

²Max Planck Institute for Intelligent Systems
Tübingen

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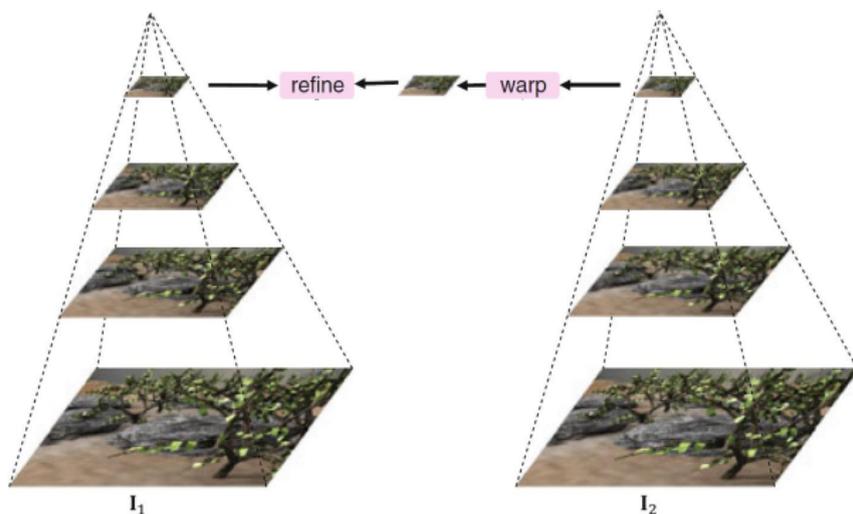
Where is Sintel going?



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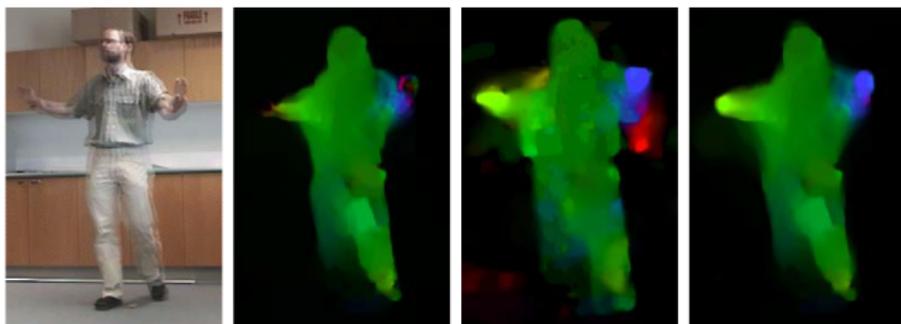


Related Work - Variational Methods

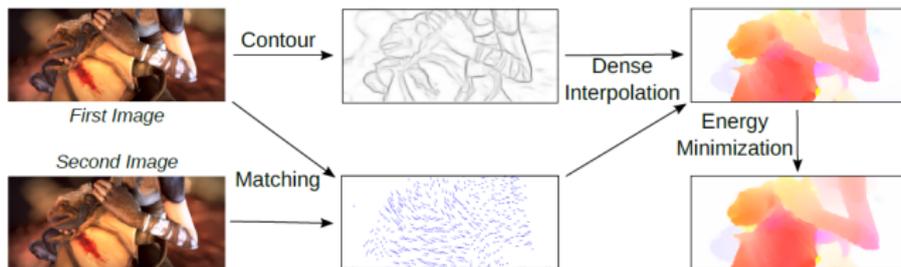


[Brox et al., ECCV 2004], [Sun et al., IJCV 2013]

Related Work - Sparse Features



[Brox et al., PAMI 2011]



[Revaud et al., CVPR 2015]

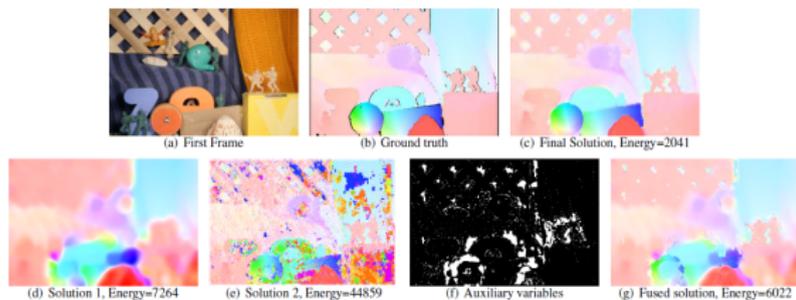
Related Work - Stereo Matching

KITTI Stereo Leaderboard

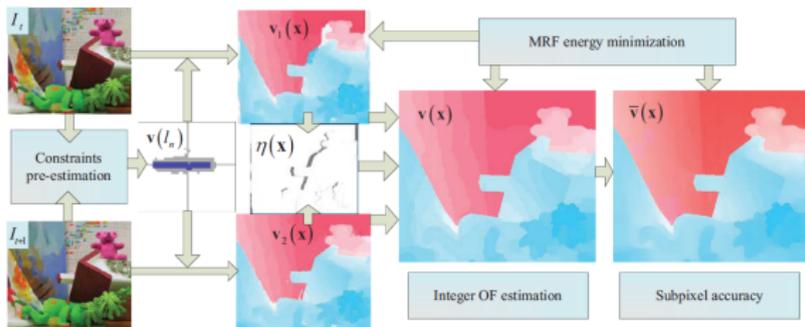
| | Method | Setting | Code | Out-Hoc | Out-All | Avg-Hoc | Avg-All | Density | Runtime | Environment | Compare |
|---|------------|---------|------|---------|---------|---------|---------|----------|---------|-------------------------------------|--------------------------|
| 1 | MC-CNN | | | 2.43 % | 3.63 % | 0.7 px | 0.9 px | 100.00 % | 120 s | Intel® GTX Titan (CUDA, Lua+Torch) | <input type="checkbox"/> |
| J. Zhang and F. LuCam | | | | | | | | | | | |
| 2 | Hybrid | | code | 2.47 % | 3.27 % | 0.7 px | 0.9 px | 100.00 % | 265 s | i8 cores @ 3.0 GHz (Matlab + C/C++) | <input type="checkbox"/> |
| F. Sauer and A. Singer: <i>Statistical Ranking System for Stereo Matching</i> , <i>ICCV2015 Workshop: Conference on Computer Vision and Pattern Recognition (CVPR) 2015</i> . | | | | | | | | | | | |
| 3 | MC-CNN | | | 2.41 % | 3.64 % | 0.8 px | 1.0 px | 100.00 % | 100 s | Intel® GTX Titan (CUDA, Lua+Torch) | <input type="checkbox"/> |
| J. Zhang and F. LuCam: <i>Computing the Stereo Matching Cost with a Conditional Neural Network</i> , <i>Conference on Computer Vision and Pattern Recognition (CVPR) 2015</i> . | | | | | | | | | | | |
| 4 | PSM | | code | 2.70 % | 3.00 % | 0.7 px | 0.7 px | 100.00 % | 300 s | 1 core @ 2.5 GHz (C/C++) | <input type="checkbox"/> |
| C. Vogel, K. Schindler and S. Roth: <i>3D Stereo Flow Estimation with a Fast and Accurate Depth Space Model</i> , <i>CV 2015</i> | | | | | | | | | | | |
| 5 | SPS-S01 | | | 2.83 % | 3.64 % | 0.8 px | 0.9 px | 100.00 % | 35 s | 1 core @ 3.5 GHz (C/C++) | <input type="checkbox"/> |
| X. Yang, H. A. Alavi and R. Urtasun: <i>Efficient Joint Segmentation, Occlusion Labeling, Stereo and Flow Estimation</i> , <i>ICCV 2014</i> . | | | | | | | | | | | |
| 6 | VC-SP | | | 3.05 % | 3.31 % | 0.8 px | 0.8 px | 100.00 % | 300 s | 1 core @ 2.5 GHz (C/C++) | <input type="checkbox"/> |
| C. Vogel, S. Roth and K. Schindler: <i>Fast Correlation 3D Stereo Flow Estimation from Multiple Frames</i> , <i>Proceedings of European Conference on Computer Vision, Lecture Notes in Computer Science 2014</i> . | | | | | | | | | | | |
| 7 | Deep Embed | | | 3.50 % | 4.24 % | 0.9 px | 1.1 px | 100.00 % | 3 s | 1 core @ 2.5 GHz (C/C++) | <input type="checkbox"/> |
| S. Chen, S. Sun, F. He, L. Wang and C. Huang: <i>A Deep Neural Network-Based Embedding Model for Stereo Matching Costs</i> , <i>ICCV 2015</i> . | | | | | | | | | | | |
| 8 | GSDF | | | 3.95 % | 3.94 % | 0.8 px | 0.9 px | 100.00 % | 120 s | 8 cores @ 2.5 GHz (C/C++) | <input type="checkbox"/> |
| Anonymous submission | | | | | | | | | | | |
| 9 | DF | | code | 3.20 % | 4.07 % | 0.8 px | 0.9 px | 99.98 % | 50 min | 1 core @ 3.0 GHz (Matlab + C/C++) | <input type="checkbox"/> |
| B. Hesse and A. Singer: <i>Object-Based Flow for Subpixel Accuracy</i> , <i>Conference on Computer Vision and Pattern Recognition (CVPR) 2015</i> . | | | | | | | | | | | |
| 10 | CoB | | code | 3.30 % | 4.10 % | 0.8 px | 0.9 px | 100.00 % | 6 s | 6 cores @ 3.3 GHz (Matlab + C/C++) | <input type="checkbox"/> |
| A. Chabert, T. Hong, S. Garcia and T. Durrant-Wise: <i>Kernel for Correlation in a Sparse Hierarchy of Images</i> , <i>CVPR 2015</i> . | | | | | | | | | | | |

- ▶ Naïve discretization of 2D flow space intractable
- ▶ 3 strategies enable discrete optimization for optical flow

Related Work - Discrete Optimization



[Lempitzky et al., CVPR 2008]



[Mozerov, TIP 2013]

Discrete Optical Flow

Optical flow as labeling task

$$E(\mathbf{l}) = \sum_{\mathbf{p} \in \mathcal{P}} \underbrace{\varphi_{\mathbf{p}}(l_{\mathbf{p}})}_{\text{data}} + \sum_{\mathbf{p} \sim \mathbf{q}} \underbrace{\psi_{\mathbf{p}, \mathbf{q}}(l_{\mathbf{p}}, l_{\mathbf{q}})}_{\text{smoothness}}$$

- ▶ \mathbf{p}, \mathbf{q} : pixels
- ▶ l : discrete flow label

Discrete Optical Flow

- ▶ Robust data term based on DAISY descriptors \mathbf{d}

$$\varphi_{\mathbf{p}}(l_{\mathbf{p}}) = \min \left(\|\mathbf{d}_{\mathbf{p}} - \mathbf{d}'_{\mathbf{p}}(l_{\mathbf{p}})\|_1, \tau_{\varphi} \right)$$

- ▶ Similar flow vectors \mathbf{f} are encouraged by

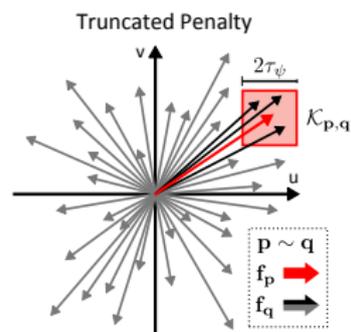
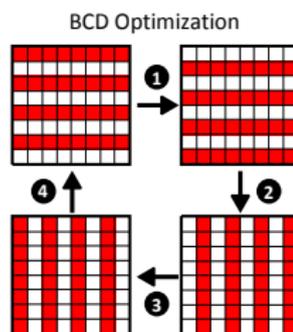
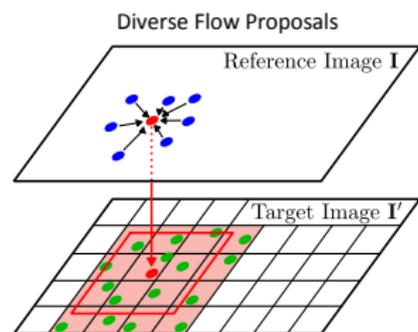
$$\psi_{\mathbf{p},\mathbf{q}}(l_{\mathbf{p}}, l_{\mathbf{q}}) = w_{\mathbf{p},\mathbf{q}} \cdot \min \left(\|\mathbf{f}_{\mathbf{p}}(l_{\mathbf{p}}) - \mathbf{f}_{\mathbf{q}}(l_{\mathbf{q}})\|_1, \tau_{\psi} \right)$$

weighted by the edge strength [Dollár et al., ICCV 2013]:

$$w_{\mathbf{p},\mathbf{q}} = \exp \left(-\alpha \kappa_{\mathbf{p},\mathbf{q}}^2 \right)$$

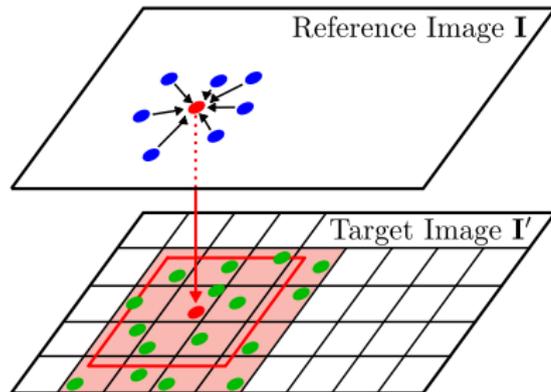
Strategies

Three strategies for efficient inference:



1. Diverse Flow Proposals

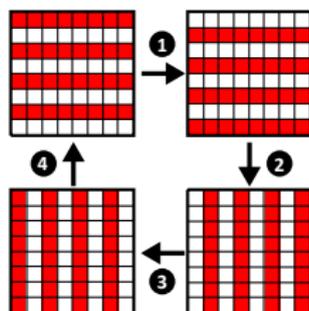
- ▶ Efficient search structure
- ▶ 300 nearest neighbors
- ▶ 200 proposals from neighboring pixels



- ▶ Reduction of proposals from 250,000 to 500 per pixel

2. Block Coordinate Descent

- ▶ Inference via Block Coordinate Descent (BCD)¹

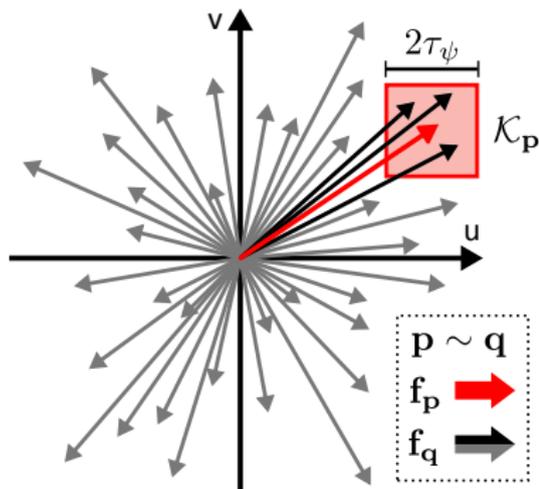


- ▶ Alternating optimization of image rows and columns
- ▶ Sub-problems solved optimally via dynamic programming

¹Chen and Koltun, CVPR 2014

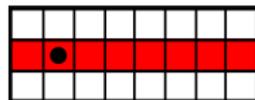
3. Truncated Pairwise Potential

- ▶ Evaluation of only a few combinations



3. Truncated Pairwise Potential

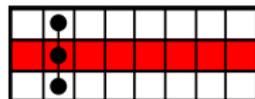
Naïve Dynamic Programming:



$$\mathbf{C}(x, l) = \varphi_{(x,y)}(l)$$

3. Truncated Pairwise Potential

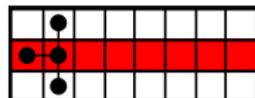
Naïve Dynamic Programming:



$$\begin{aligned} \mathbf{C}(x, l) &= \varphi_{(x,y)}(l) \\ &+ \psi_{(x,y),(x,y-1)}(l, l_{x,y-1}^*) \\ &+ \psi_{(x,y),(x,y+1)}(l, l_{x,y+1}^*) \end{aligned}$$

3. Truncated Pairwise Potential

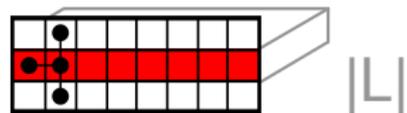
Naïve Dynamic Programming:



$$\begin{aligned} \mathbf{C}(x, l) = & \varphi_{(x,y)}(l) \\ & + \psi_{(x,y),(x,y-1)}(l, l_{x,y-1}^*) \\ & + \psi_{(x,y),(x,y+1)}(l, l_{x,y+1}^*) \\ & + \min_{0 \leq k < L} (\psi_{(x,y),(x-1,y)}(l, k) + \mathbf{C}(x-1, k)) \end{aligned}$$

3. Truncated Pairwise Potential

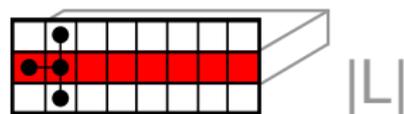
Naïve Dynamic Programming:



$$\begin{aligned} \mathbf{C}(x, l) = & \varphi_{(x,y)}(l) \\ & + \psi_{(x,y),(x,y-1)}(l, l_{x,y-1}^*) \\ & + \psi_{(x,y),(x,y+1)}(l, l_{x,y+1}^*) \\ & + \min_{0 \leq k < L} (\psi_{(x,y),(x-1,y)}(l, k) + \mathbf{C}(x-1, k)) \end{aligned}$$

3. Truncated Pairwise Potential

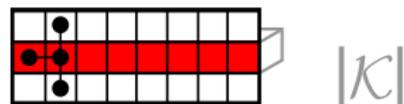
Efficient Dynamic Programming:



$$\begin{aligned} \mathbf{C}(x, l) &= \varphi_{(x,y)}(l) \\ &+ \psi_{(x,y),(x,y-1)}(l, l_{x,y-1}^*) \\ &+ \psi_{(x,y),(x,y+1)}(l, l_{x,y+1}^*) \end{aligned}$$

3. Truncated Pairwise Potential

Efficient Dynamic Programming:

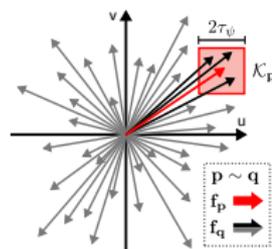


$$\begin{aligned} \mathbf{C}(x, l) &= \varphi_{(x,y)}(l) \\ &\quad + \psi_{(x,y),(x,y-1)}(l, l_{x,y-1}^*) \\ &\quad + \psi_{(x,y),(x,y+1)}(l, l_{x,y+1}^*) \end{aligned}$$

$$+ \min \left(\min_{k \in \mathcal{K}_{(x,y),(x-1,y),l}} (\psi_{(x,y),(x-1,y)}(l, k) + \mathbf{C}(x-1, k)), c \right)$$

3. Truncated Pairwise Potential

Efficient Dynamic Programming:



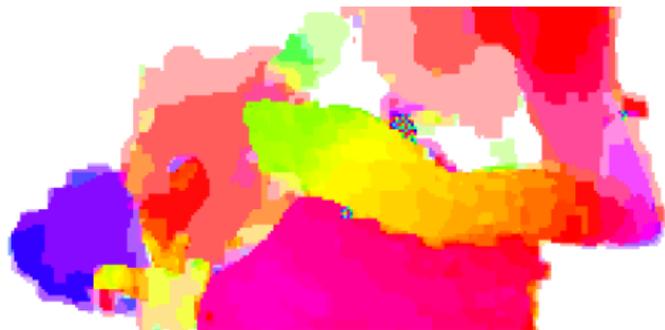
$$\begin{aligned} \mathbf{C}(x, l) = & \varphi_{(x,y)}(l) \\ & + \psi_{(x,y),(x,y-1)}(l, l_{x,y-1}^*) \\ & + \psi_{(x,y),(x,y+1)}(l, l_{x,y+1}^*) \end{aligned}$$

$$+ \min \left(\min_{k \in \mathcal{K}_{(x,y),(x-1,y),l}} (\psi_{(x,y),(x-1,y)}(l, k) + \mathbf{C}(x-1, k)), c \right)$$

$$\text{with: } c = \min_{0 \leq k < L} (w_{(x,y),(x-1,y)} \tau_\varphi + \mathbf{C}(x-1, k))$$

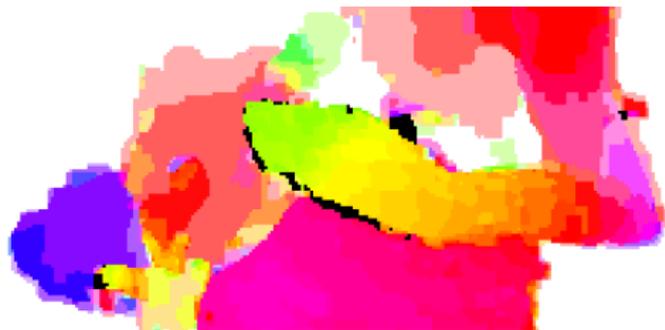
Post-Processing

- ▶ Inference yields integral flow



Post-Processing

- ▶ Inference yields integral flow
- ▶ Forward-backward consistency check



Post-Processing

- ▶ Inference yields integral flow
- ▶ Forward-backward consistency check
- ▶ Interpolation & subpixel refinement
 - ▶ Epicflow [Revaud, CVPR 2015]



Quantitative Evaluation - MPI Sintel

| | EPE (px) | | |
|--|--------------|--------------|---------------|
| | All | Noc | Occ |
| Flow Fields [Bailer et al., ICCV 2015] | 5.810 | 2.621 | 31.799 |
| Ours+EpicFlow | 6.077 | 2.937 | 31.685 |
| DM+EpicFlow [Revaud et al., CVPR 2015] | 6.285 | 3.060 | 32.564 |
| TF+OFM [Kennedy et al., EMMCVPR 2015] | 6.727 | 3.388 | 33.929 |
| Deep+R [Drayer et al., BMVC 2015] | 6.769 | 2.996 | 37.494 |

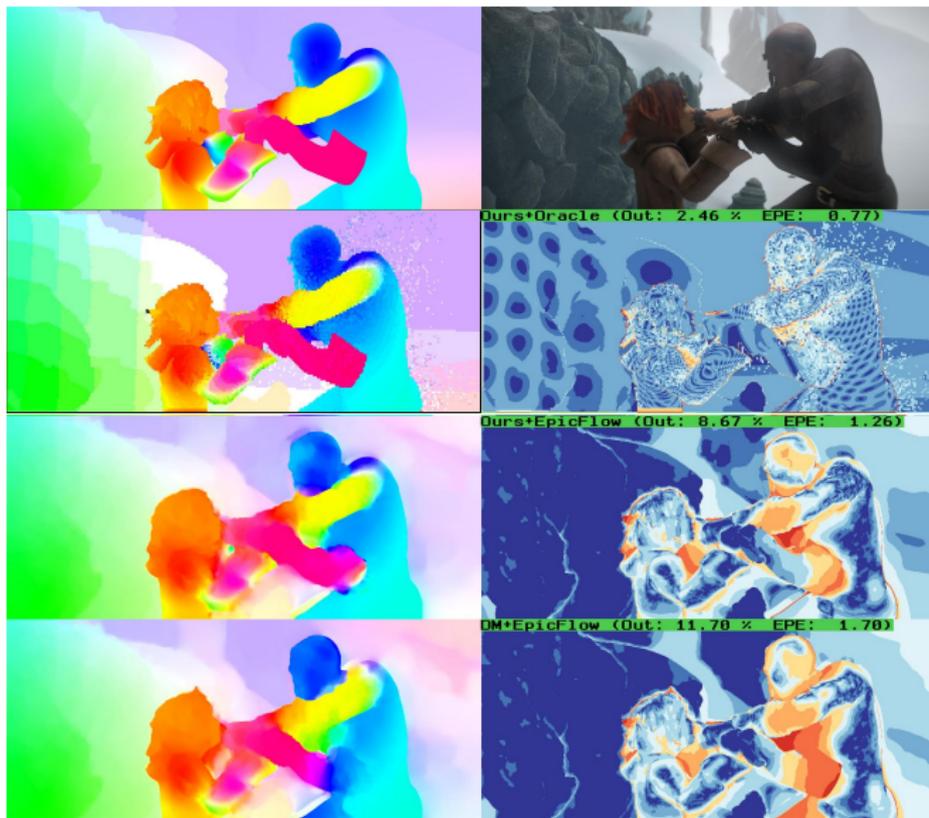
MPI Sintel final (48 methods listed online)

Quantitative Evaluation - MPI Sintel

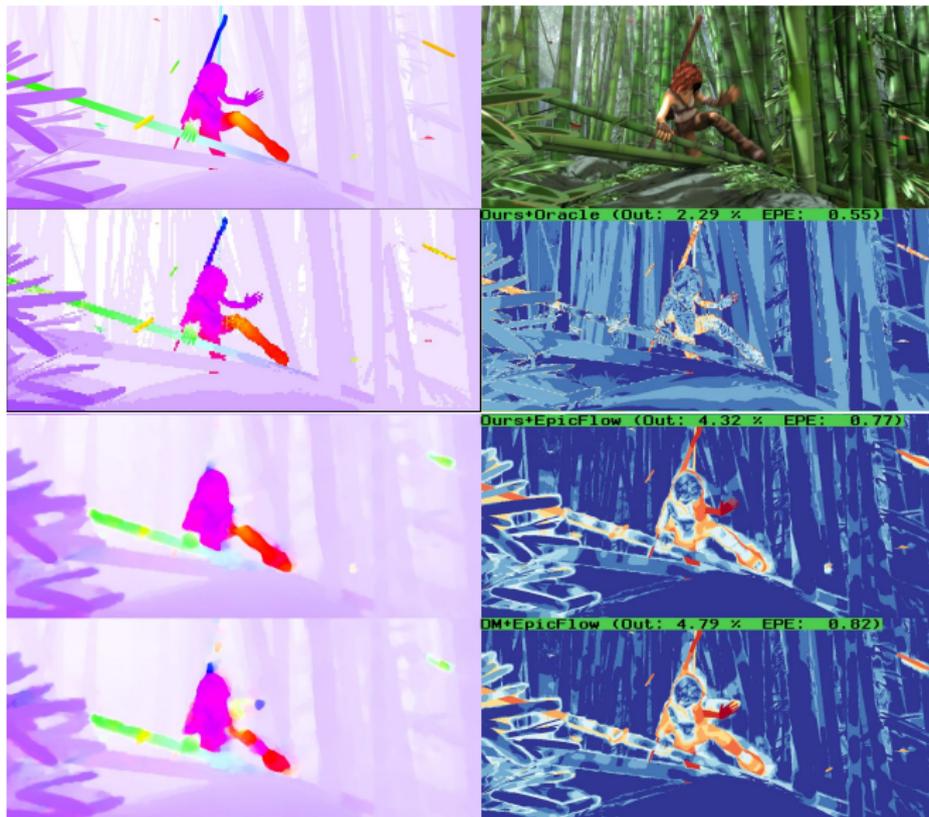
| | EPE (px) | | |
|--|--------------|--------------|---------------|
| | All | Noc | Occ |
| Ours+EpicFlow | 3.567 | 1.108 | 23.626 |
| FlowFields [Bailer et al., ICCV 2015] | 3.748 | 1.056 | 25.700 |
| DM+EpicFlow [Revaud et al., CVPR 2015] | 4.115 | 1.360 | 26.595 |
| PH-Flow [Yang et al., CVPR 2015] | 4.388 | 1.714 | 26.202 |
| AggregFlow [Fortun et al., ARXIV 2014] | 4.754 | 1.694 | 29.685 |

MPI Sintel clean (48 methods listed online)

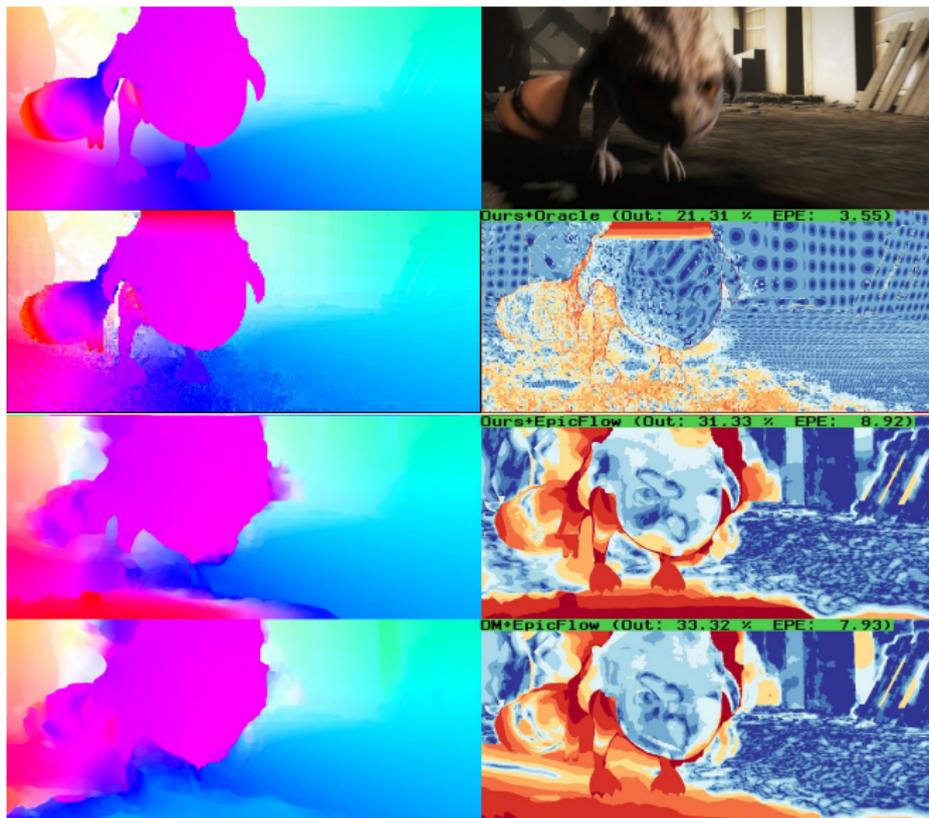
Qualitative Results - MPI Sintel



Qualitative Results - MPI Sintel



Qualitative Results - MPI Sintel



Quantitative Evaluation - KITTI

| | Outliers (%) | | EPE (px) | |
|---|--------------|--------------|------------|------------|
| | Noc | All | Noc | All |
| PH-Flow [Yang et al., CVPR 2015] | 5.76 | 10.57 | 1.3 | 2.9 |
| FlowFields [Bailer et al., ICCV 2015] | 5.77 | 14.01 | 1.4 | 3.5 |
| NLTGV-SC [Ranftl et al., ECCV 2014] | 5.93 | 11.96 | 1.6 | 3.8 |
| DDS-DF [Wei et al., 3DV 2014] | 6.03 | 13.08 | 1.6 | 4.2 |
| TGV2ADCSIFT [Braux-Zin et al., ICCV 2013] | 6.20 | 15.15 | 1.5 | 4.5 |
| Ours+EpicFlow | 6.23 | 16.63 | 1.3 | 3.6 |
| AnyFlow [Submitted to PAMI] | 6.37 | 15.80 | 1.5 | 4.3 |
| BTF-ILLUM [Demetz et al., ECCV 2014] | 6.52 | 11.03 | 1.5 | 2.8 |
| CRT-TGV [Submitted to IJCV] | 6.71 | 12.09 | 2.0 | 3.9 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| DM+EpicFlow [Revaud et al., CVPR 2015] | 7.88 | 17.08 | 1.5 | 3.8 |

KITTI 3 px (69 methods listed online)

Quantitative Evaluation - KITTI

| | Outliers (%) | | EPE (px) | |
|---|--------------|-------------|------------|------------|
| | Noc | All | Noc | All |
| Ours+EpicFlow | 3.89 | 12.46 | 1.3 | 3.6 |
| PH-Flow [Yang et al., CVPR 2015] | 3.93 | 7.72 | 1.3 | 2.9 |
| FlowFields [Bailer et al., ICCV 2015] | 3.95 | 10.21 | 1.4 | 3.5 |
| DDS-DF [Wei et al., 3DV 2014] | 4.41 | 10.41 | 1.6 | 4.2 |
| NLTGV-SC [Ranftl et al., ECCV 2014] | 4.50 | 9.42 | 1.6 | 3.8 |
| AnyFlow [Submitted to PAMI] | 4.51 | 12.55 | 1.5 | 4.3 |
| TGV2ADCSIFT [Braux-Zin et al., ICCV 2013] | 4.60 | 12.17 | 1.5 | 4.5 |
| BTF-ILLUM [Demetz et al., ECCV 2014] | 4.64 | 8.11 | 1.5 | 2.8 |
| CRT-TGV [Submitted to IJCV] | 5.01 | 8.97 | 2.0 | 3.9 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| DM+EpicFlow [Revaud et al., CVPR 2015] | 5.36 | 12.86 | 1.5 | 3.8 |

KITTI 5 px (69 methods listed online)

Conclusions

- ▶ Three strategies to reduce computational complexity
- ▶ Enable discrete optimization for optical flow
- ▶ State-of-the-art performance with sub-pixel refinement

Outlook

- ▶ Integrate multiple frames
- ▶ Reason about multiple scales
- ▶ Include semantic knowledge

Discrete Optimization for Optical Flow

Thank you!



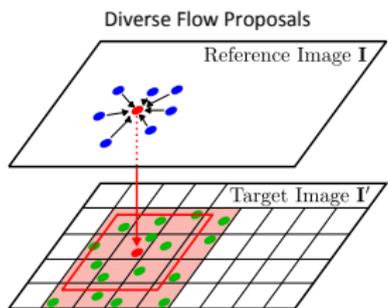
Project page:

www.cvlibs.net/projects/discrete_flow

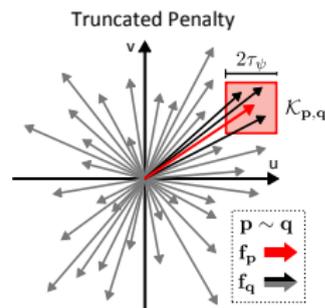
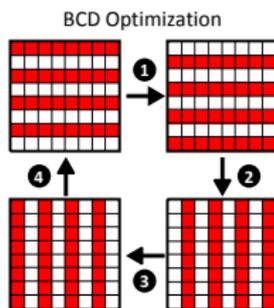
Runtimes

Percentage Runtime:

40%



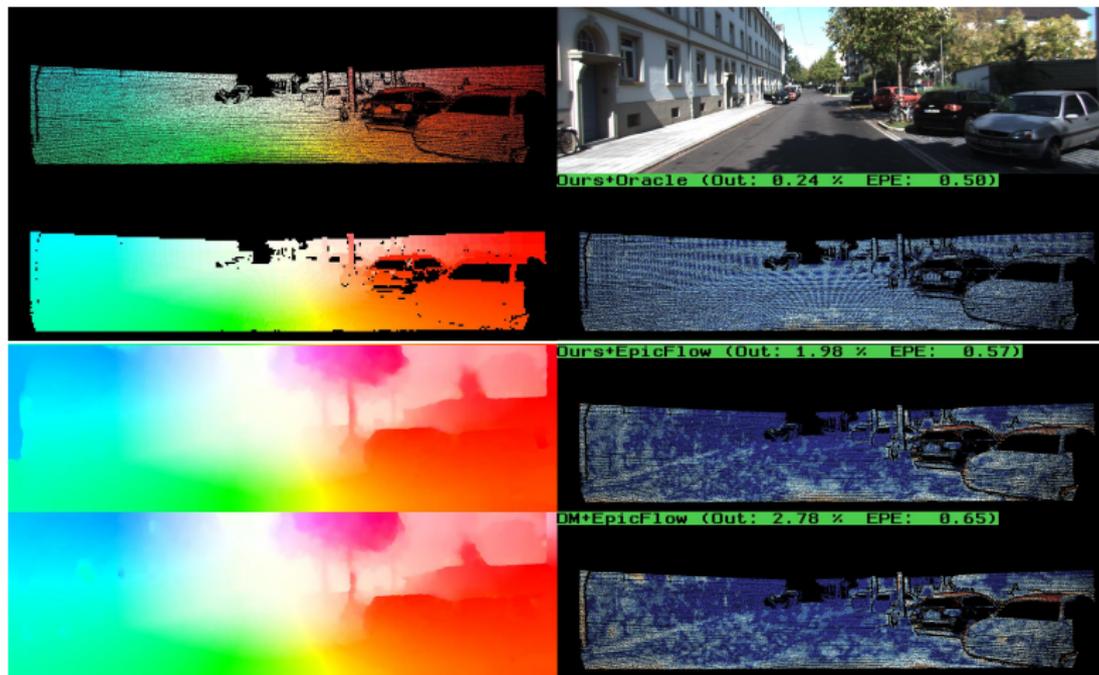
35%



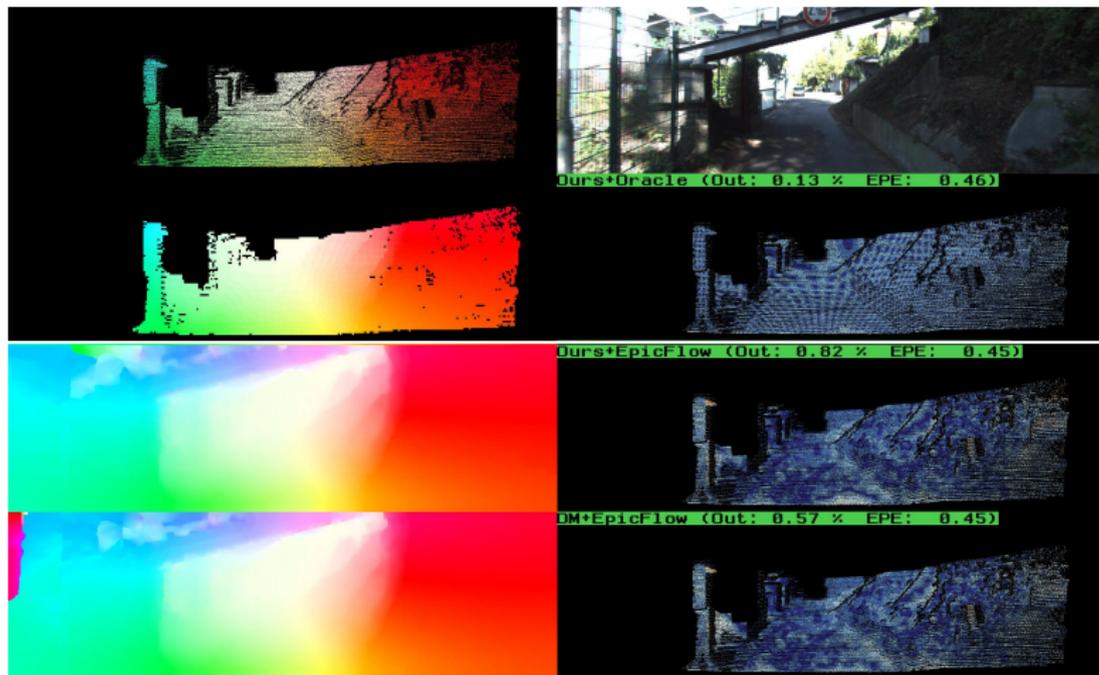
(remaining 25% for descriptor extraction and post-processing)

- ▶ Total runtime ~ 3 minutes

Qualitative Results - KITTI



Qualitative Results - KITTI



Qualitative Results - KITTI

